Azonosító jel: | | | | | | | | | | | | | | |

ÉRETTSÉGI VIZSGA ● 2011. május 3.

MATEMATIKA ANGOL NYELVEN

EMELT SZINTŰ ÍRÁSBELI VIZSGA

2011. május 3. 8:00

Az írásbeli vizsga időtartama: 240 perc

Pótlapok száma	
Tisztázati	
Piszkozati	

NEMZETI ERŐFORRÁS MINISZTÉRIUM

Instructions to candidates

1. The time allowed for this examination paper is 240 minutes. When that time is over, you will have to stop working.

2. You may solve the problems in any order.

3. In Section II, you are only required to solve four out of the five problems**. When you have finished the examination, write in the square below the number of the problem NOT selected.**
   If it is not clear for the examiner which problem you do not want to be assessed, then problem 9 will not be assessed.

4. In solving the problems, you are allowed to use a calculator that cannot store and display verbal information. You are also allowed to use any book of four-digit data tables. The use of any other electronic devices, or printed or written material is forbidden.

5. **Always write down the reasoning used in obtaining the answers, since a large part of the attainable points will be awarded for that.**

6. **Make sure that the calculations of intermediate results are also possible to follow.**

7. In solving the problems, theorems studied and given a name in class (e.g. the Pythagorean theorem or the altitude theorem) do not need to be stated precisely. It is enough to refer to them by the name, but their applicability needs to be briefly explained. Reference to other theorem(s) will only be awarded full mark if the theorem and all its conditions are stated correctly (proof is not required), and the applicability of the theorem to the given problem is explained.

8. Always state the final result (the answer to the question of the problem) in words, too.

9. Write in pen. The examiner is instructed not to mark anything in pencil, other than diagrams. Diagrams are allowed to be drawn in pencil. If you cancel any solution or part of a solution by crossing it over, it will not be assessed.

10. Only one solution to each problem will be assessed. In the case of more than one attempt to solve a problem**, indicate clearly** which attempt you wish to be marked.

11. Do not write anything in the grey rectangles.

I.

1. Consider the function $f:[-2;5]\to \mathbf{R}$, $f(x)=-x^2-2x+3$.

a) Describe the function: Where does it increase or decrease, where does it have a maximum or minimum, and what is the maximum or minimum value?

b) Find the largest subset of the interval $[-2;5]$ on which the expression $g(x)=\dfrac{1}{\lg(x^2+2x-3)-\lg 5}$ is meaningful.

a)	7 points	
b)	7 points	
T.:	14 points	

2. 29 students of a university applied to attend a postgraduate course. They were asked whether they had language certificates in English, German or French. Each applicant answered each of the three questions (with yes or no).
It turned out that 22 of the applicants had certificates in English, 18 had German and 18 had French. 12 students had certificates in both German and French, but 3 of these did not have a certificate in English. Out of those with certificates in English, 7 did not have a certificate in German and 8 did not have a certificate in French.

a) How many applicants answered "yes" to all three questions?

b) How many applicants answered "no" to all three questions?

a)	3 points	
b)	9 points	
T.:	12 points	

3. A fruit and vegetable retailer bought 165 kg of apricots at the wholesale market on Monday. On Tuesday, he bought 165 kg of peaches. One crate of peaches weigh 2 kg less than one crate of apricots, thus there were 8 crates more peaches than apricots.
How many kg of apricots did a crate contain, and how many crates of apricots did the merchant buy on Monday? (Each crate contained the same number of kg of apricots on Monday, and each contained the same number of kg of peaches on Tuesday.)

T.:	12 points	

4. The base of a regular four-sided pyramid *ABCDE* is the square *ABCD*. The pyramid is cut with the plane *EAC*. The area of the intersection is 64 cm^2. If the pyramid is cut with a plane parallel to the base at a distance of 4 cm from vertex *E*, the area of the intersection will be 32 cm^2.

a) Find the height of the pyramid, and find the area of the base.

b) Calculate the angle enclosed by the base and a lateral face.

a)	10 points	
b)	3 points	
T.:	13 points	

II.

You are required to solve any four out of the problems 5 to 9. Write the number of the problem NOT selected in the blank square on page 3.

5. Consider the sequence whose nth term is $a_n = n + \sin(n\alpha)$, where $\alpha \in \mathbf{R}$ is a given number.

a) Let $\alpha = \frac{\pi}{3}$. Write down the exact values of the first three terms of the sequence.

b) For what values of $\alpha \in [0; 2\pi]$ will the numbers a_1, a_2, a_3 számok, in this order, be three consecutive terms of a non-constant arithmetic progression?

In your solution, you may use the following identities:

$$\sin\alpha + \sin\beta = 2\sin\frac{\alpha+\beta}{2}\cdot\cos\frac{\alpha-\beta}{2};$$

$$\sin 3\alpha = 3\sin\alpha - 4\sin^3\alpha.$$

a)	3 points	
b)	13 points	
T.:	16 points	

You are required to solve any four out of the problems 5 to 9. Write the number of the problem NOT selected in the blank square on page 3.

6. An urn contains one white, one red and one blue ball. Five times in a row, one ball is drawn from the urn, its colour is noted, and then the ball is replaced.

a) What is the probability that the number of blue balls and the number of red balls among the five balls drawn are equal?

b) What is the probability that more blue balls are drawn than red balls?

a)	8 points	
b)	8 points	
T.:	16 points	

You are required to solve any four out of the problems 5 to 9. Write the number of the problem NOT selected in the blank square on page 3.

7. 5% of the population of large cities become ill with a mild but contagious new disease. 45% of those catching the disease are smokers, whereas among those not catching the disease the proportion of smokers is only 20%.

a) One hundred inhabitants are chosen at random in a city. What is the probability that at least two of them become ill with this disease?
(Round your answer to two decimal places.)

b) Calculate what percentage of smokers and what percentage of non-smokers catch the disease.
(Round your answers to one decimal place.)

a)	7 points	
b)	9 points	
T.:	16 points	

You are required to solve any four out of the problems 5 to 9. Write the number of the problem NOT selected in the blank square on page 3.

8. Pali and Zoli bought a 60 m × 30 m rectangular plot of land together. They divided the plot between themselves into two congruent right-angled trapeziums. The shorter base of each trapezium is 20 m long. Let *EF* denote the line segment forming the common boundary of the two parts.

a) Calculate the length of *EF*.
(Express your answer in metres, rounded to one decimal place.)

Pali was supposed to build a fence along the common boundary, but he did not have enough money. So he made the following offer to Zoli: He would give a triangular piece of his own plot to Zoli, provided that Zoli builds the fence separating their plots. Since Zoli wished to increase the 20-metre-long side of his plot by at most 8 metres, he accepted the offer and specified that the new common boundary was to be the line segment *EG*. The price of one square metre of land is 30 000 forints, and one metre of fence costs 15 000 forints to build. Any further costs were to be divided equally between them.

b) What is the minimum possible length of the line segment *FG*, so that the deal is to Zoli's advantage?
(Round your answer to one decimal place.)

a)	4 points	
b)	12 points	
T.:	16 points	

You are required to solve any four out of the problems 5 to 9. Write the number of the problem NOT selected in the blank square on page 3.

9. A toy factory manufactures sets of wooden building blocks. If x sets are made daily, the total manufacturing costs are expressed by $c(x)=\frac{x^{1,5}}{5}+12x+300$ euros. They can sell each set for 18 euros.

a) How many sets of building blocks should the factory manufacture a day to achieve the maximum possible profit? What is the maximum profit they can make?

b) One of the building blocks in the set is made from a cube of edge 3 cm by cutting off each of the eight "corners" with a plane: Each of the three edges that meet at a vertex is cut by the plane of the saw at a distance of 1 cm from the vertex. What percentage of the volume of the original cube is the volume of the solid obtained in this way?
Round your answer to the nearest integer.
(In your calculations, ignore any loss of material owing to the sawing process.)

a)	9 points	
b)	7 points	
T.:	16 points	

	number of problem	maximum score	points awarded	maximum score	points awarded
Section I.	1.	14		**51**	
	2.	12			
	3.	12			
	4.	13			
Section II.		16		**64**	
		16			
		16			
		16			
		← problem not selected			
Total score on written examination				**115**	

date

examiner

	Elért pontszám **egész számra** kerekítve /score rounded to **integer**	Programba beírt **egész** pontszám / **integer** score entered in program
I. rész / Section I.		
II. rész / Section II.		

Javító tanár / examiner

Jegyző / registrar

Dátum / date

Dátum / date